

# ***Data-Interpolating Variational Analysis***

## ***Marine Observation and Data Expert Group***

Jean-Marie Beckers and GHER group Aida Alvera, Alexander Barth,  
Luc Vandembulcke, Charles Troupin, Damien Sirjacobs, Mohamed  
Ouberdous



<http://modb.oce.ulg.ac.be/GHER>



Université de Liège  
MARE-GHER Sart-Tilman B5  
4000 Liège, Belgique



# Outline

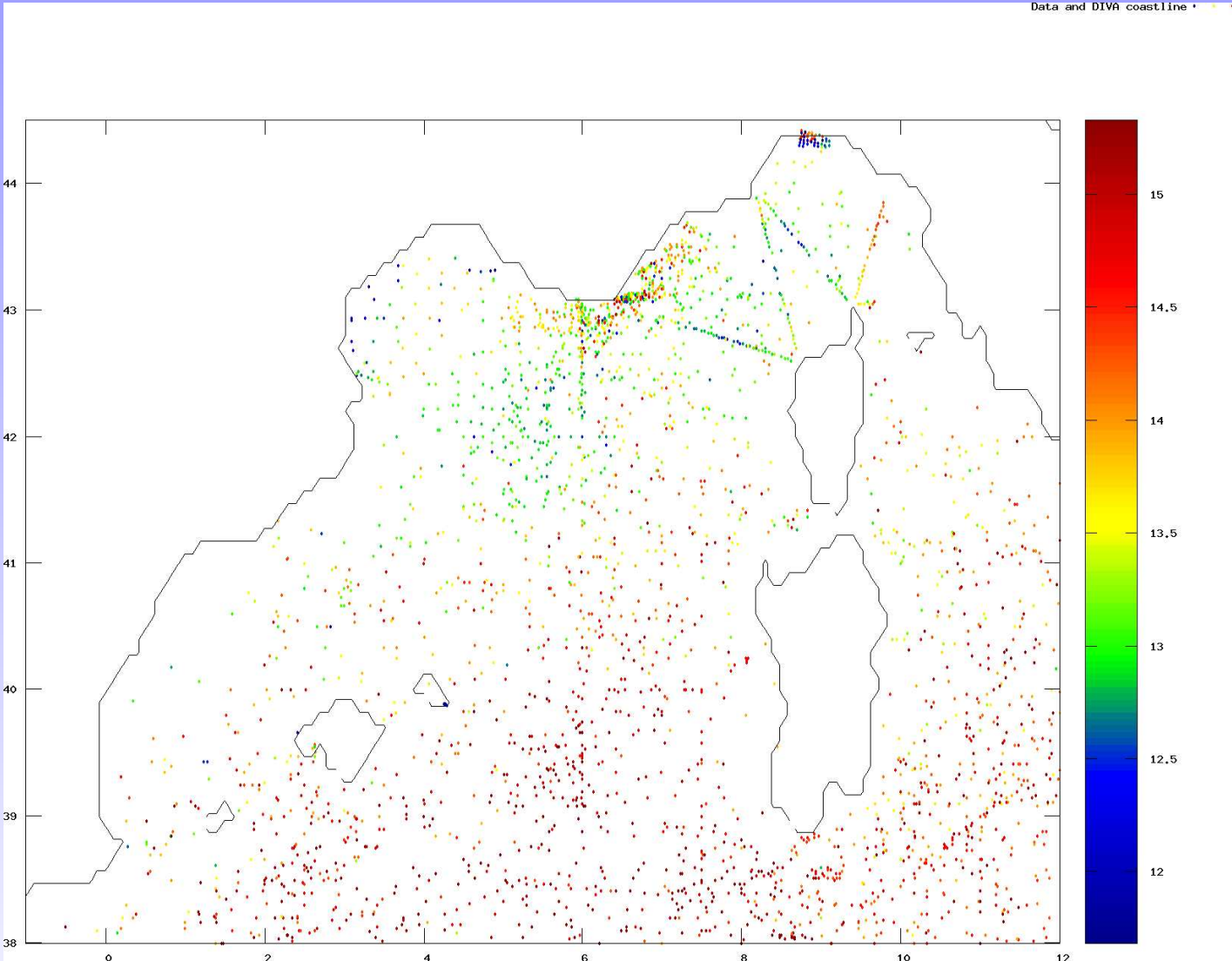
- *Basics*
- *When to use*
- *Critical points*
- *Examples*
- *Plans*



- **Basics**
- *When to use*
- *Critical points*
- *Examples*
- *Plans*



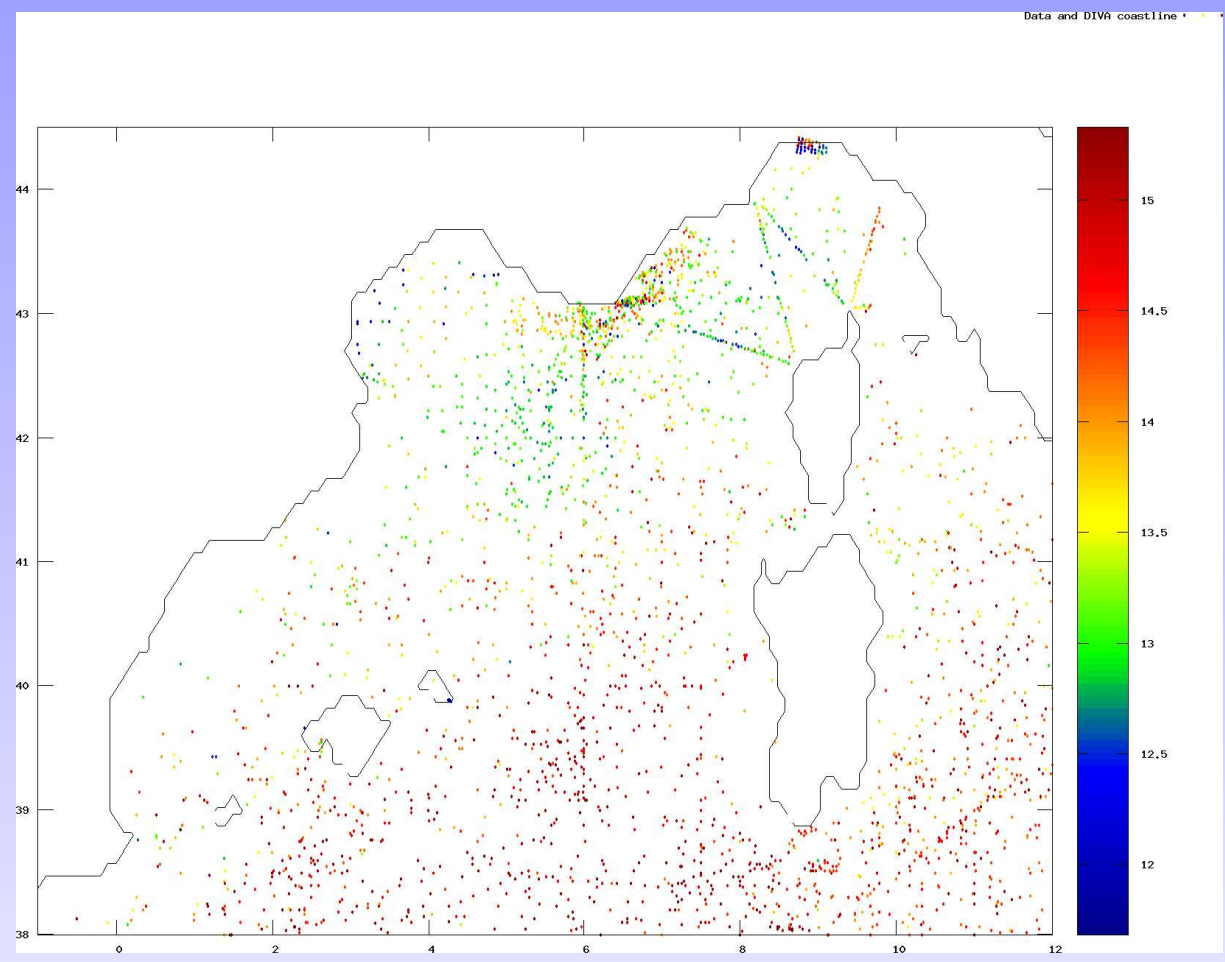
# Common problem



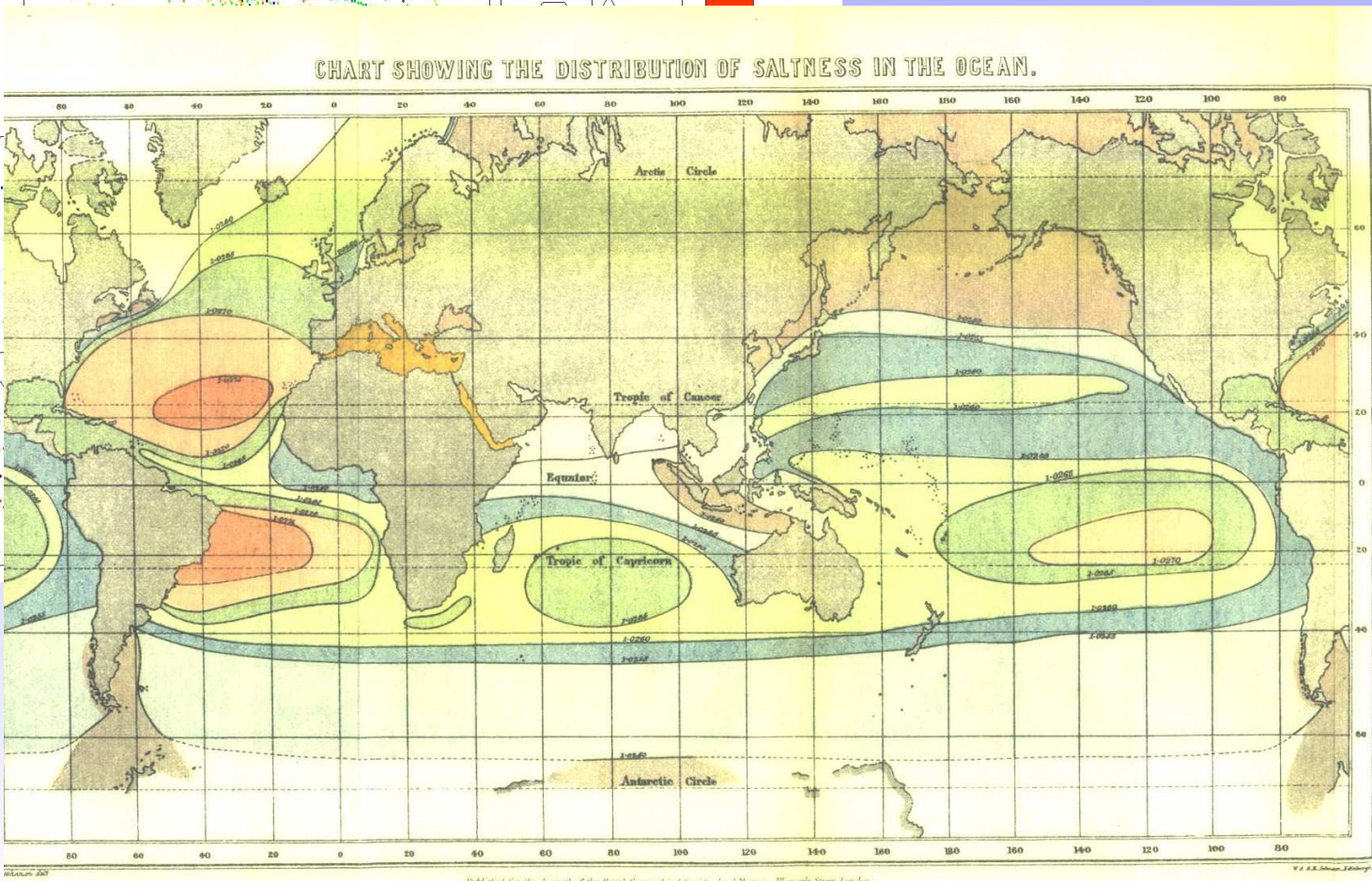
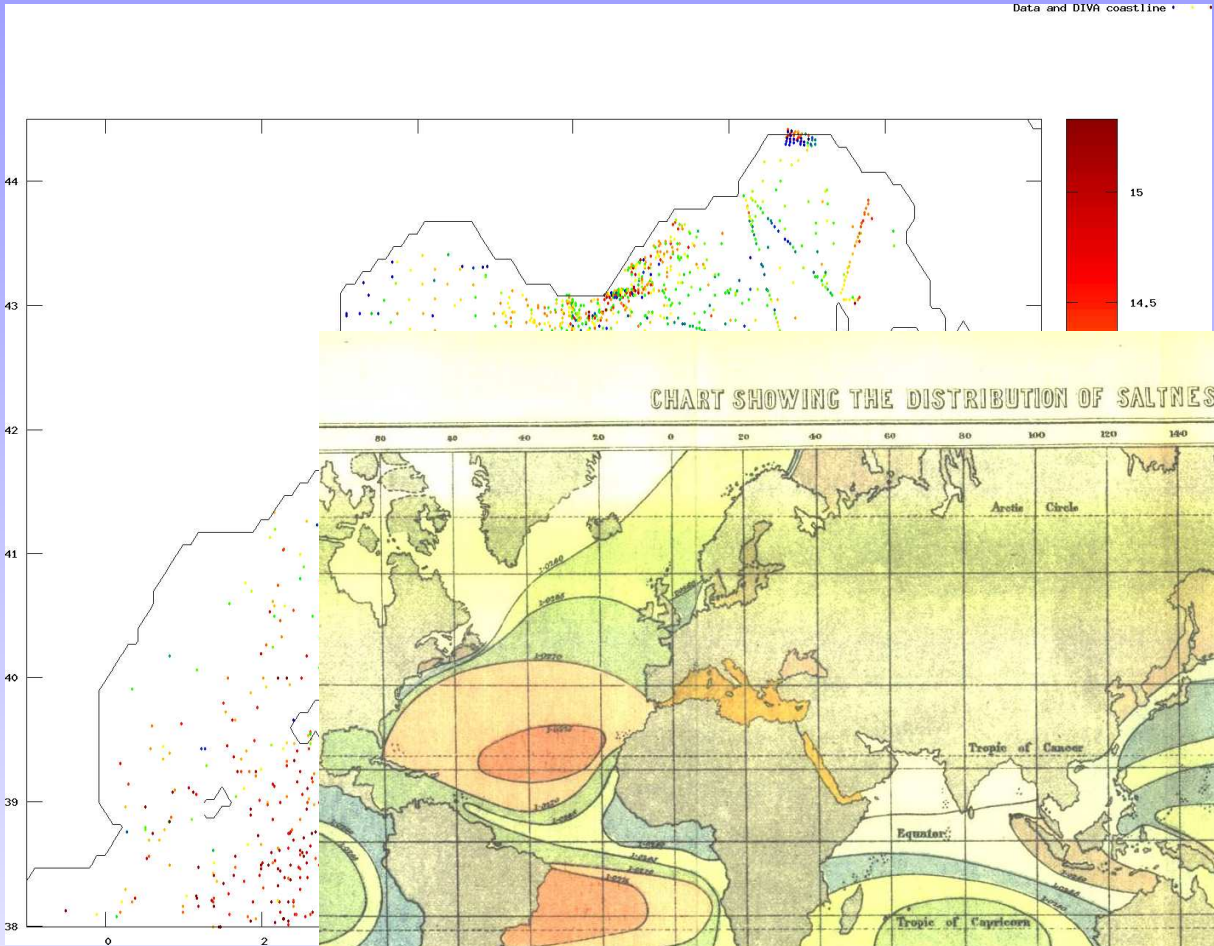
Appears when trying to produce maps, calculate volume averages, prepare initial conditions for models, quality control of data ...



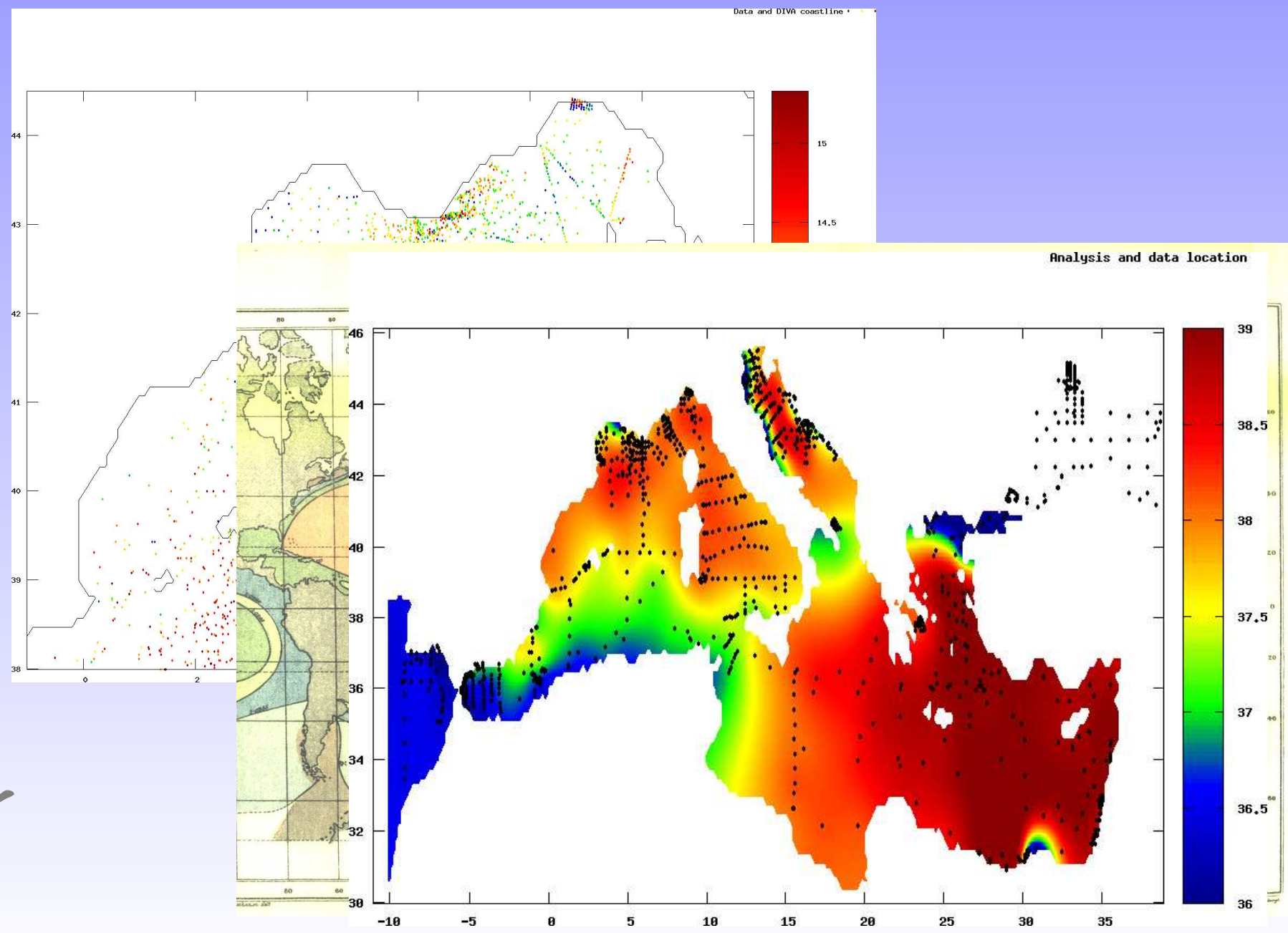
# Solutions



# Solutions



# Solutions



# Solutions



myOcean

- Project
- Products & Services
- User's Feedback

## MyOcean

myOcean

Analysis and data location

**Image of the month:**  
**ARCTIC ICE ON THE MOVE**

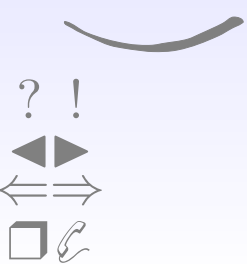
Ice extent record minima, the opening of the Northeast or Northwest Sea Passages... The Arctic is often in the news these days. Modelling and forecasting its moves as well as its reaction to climate change are important issues.

Jan

Aug

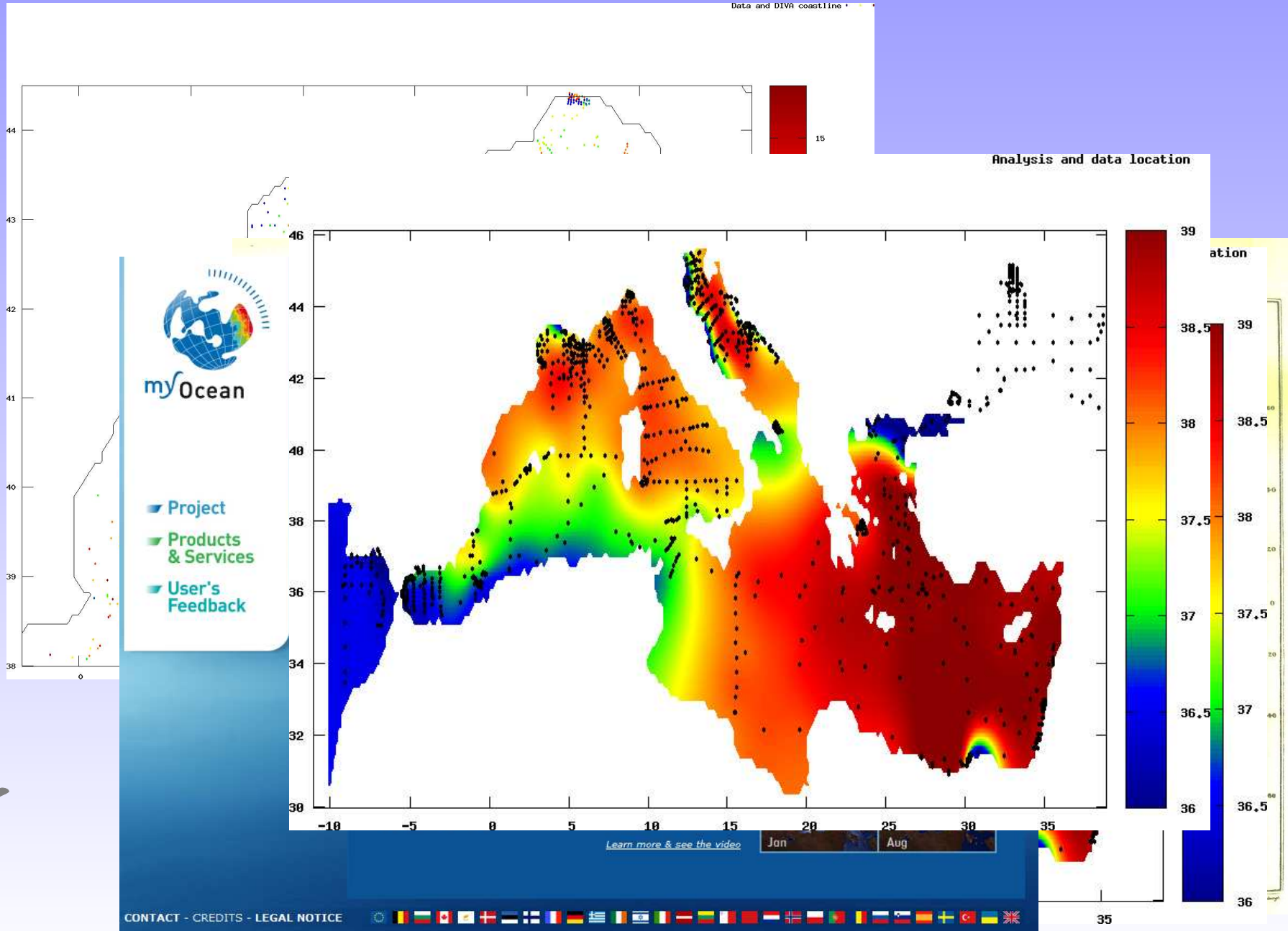
[Learn more & see the video](#)

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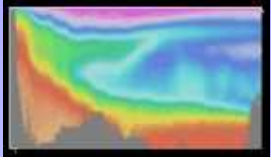
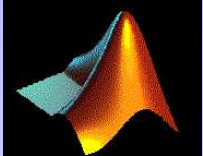






# Solutions



# Statistical spatial analysis

| Tools   | Formats                               | Method                       |
|---|---------------------------------------|------------------------------|
|    | ODV spreadsheet<br>WOCE<br>WOA<br>... | Cressmann weighting          |
|    | netCDF (toolbox), CSV ascii, ...      | Polynomial interpolation ... |
|    | CSV ascii, ...                        | Kriging, OI, ...             |
|  | ODV spreadsheet                       | Variational                  |

Mostly graphics oriented, without "oceanographic" knowledge.

# DIVA basics

Variational Inverse Method, (Brasseur *et al.*, 1996). Knowing data  $d_j$  at location  $(x_j, y_j)$ , search the field  $\varphi$  which minimizes

$$J[\varphi] = \sum_{j=1}^{Nd} \mu_j [d_j - \varphi(x_j, y_j)]^2 + \|\varphi - \varphi_b\|^2 \quad (1)$$

$$\|\varphi\| = \int_D (\alpha_2 \nabla \nabla \varphi : \nabla \nabla \varphi + \alpha_1 \nabla \varphi \cdot \nabla \varphi + \alpha_0 \varphi^2) dD \quad (2)$$

The background field  $\varphi_b$  is typically the data average value.

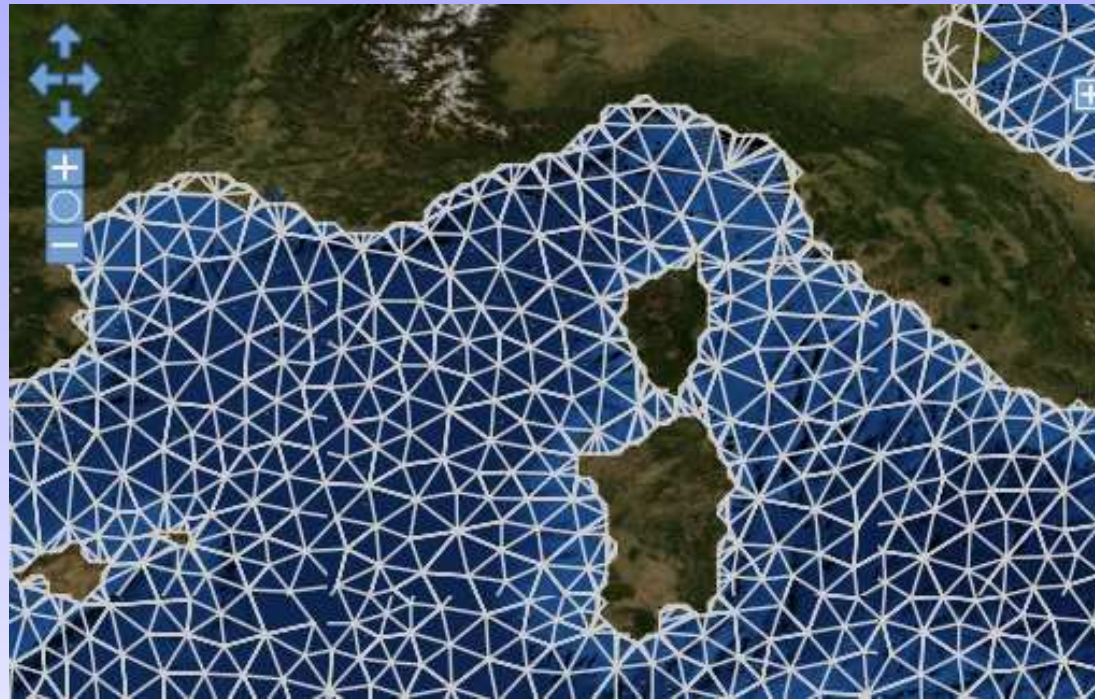
- $\alpha_0$  penalizes the field itself (anomalies),
- $\alpha_1$  penalizes gradients  $\nabla \varphi$  (no trends) ,
- $\alpha_2$  penalizes variability (regularization of second derivatives  $\nabla \nabla \varphi$ ).
- $\alpha_*$  can be related to a length scale  $L$  of the analysis.
- $\mu_j$  penalizes data-analysis misfits (objective).



# DIVA basics

$$\mu = \frac{\sigma^2}{\epsilon^2} \frac{4\pi}{L^2} \quad (3)$$

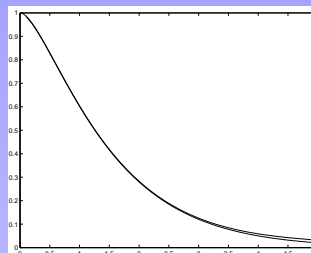
where the  $\sigma^2/\epsilon^2$  is known as a signal to noise ratio  $S/N$ .



Solution by finite element method including DIVA mesh generator using topographic data. Note decoupling of subbasins.

# DIVA as OI

DIVA is identical to the well known Optimal Interpolation



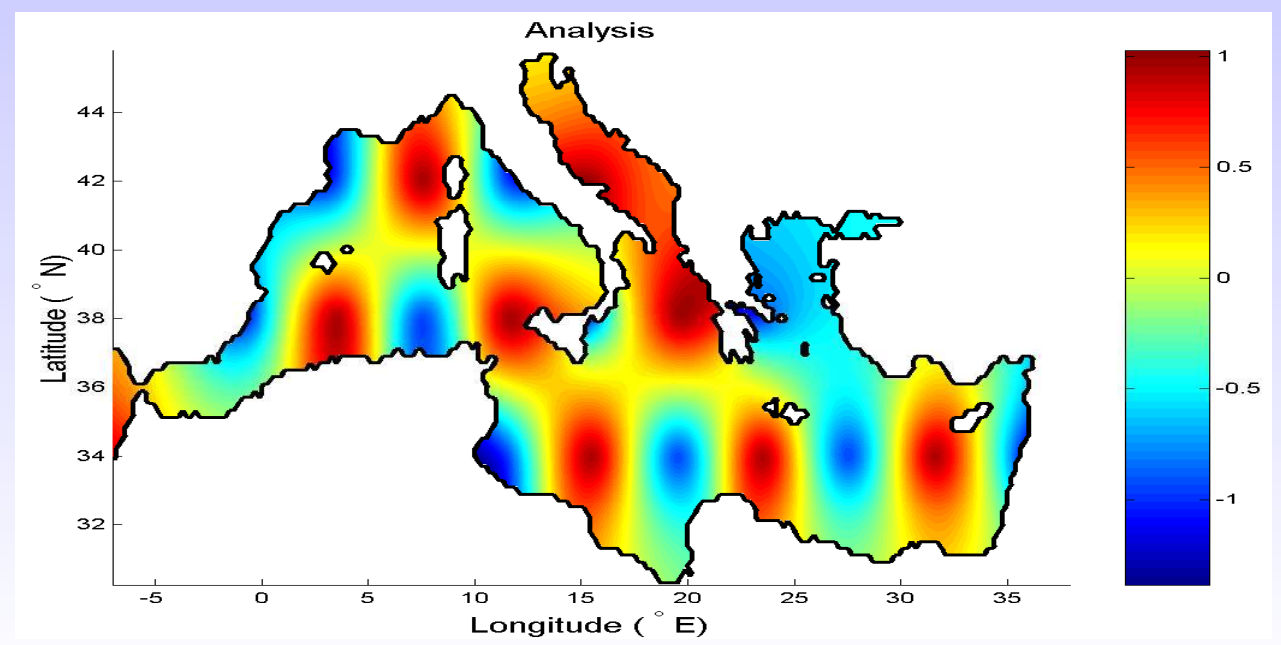
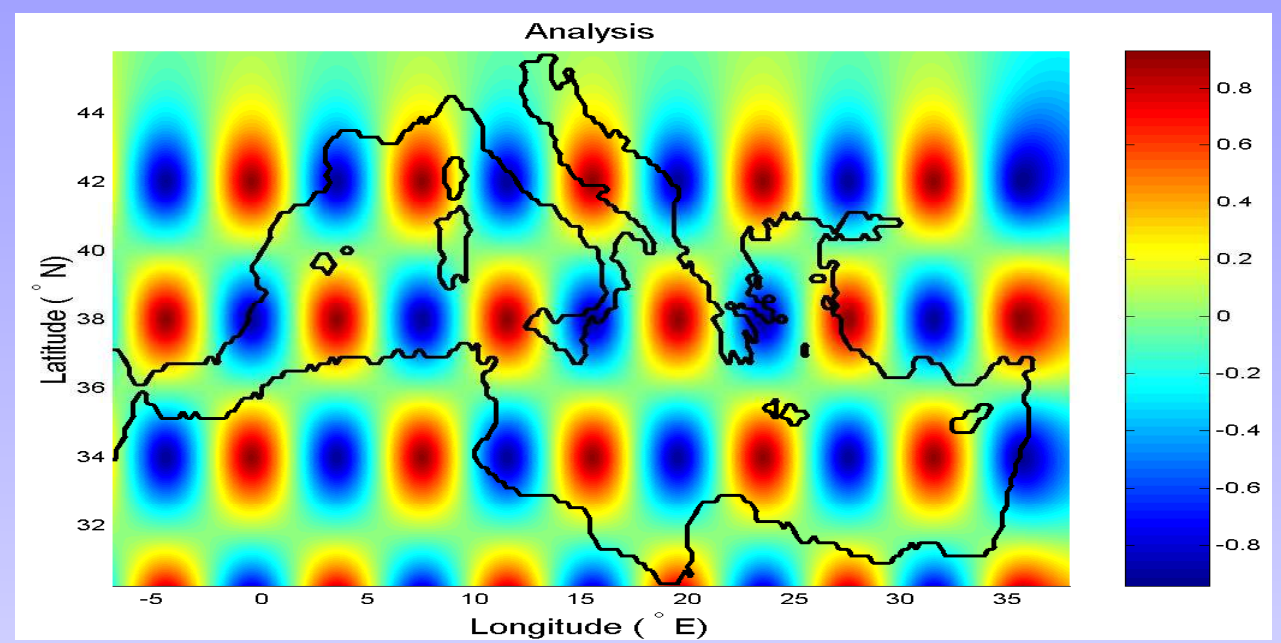
- if so-called reproducing kernel of the norm=covariance function of OI,
- if the noise is random, spatially uncorrelated and the signal/noise ratio parameter is identical.

In this case, the OI solution=DIVA solution.

- Advantages of DIVA: regularization, fast finite element solution, boundary effects taken into account.
- Difficulties: generalizations to 3D, error estimates and multivariate versions are "hybrid".

Major direct advantage of DIVA: matrix to invert is related to the finite element mesh, NOT the number of data. Useful for large data sets (Rixen *et al.*, 2000). Equivalence allows to calculate error fields with DIVA even if formulation does not rely on error minimisation.

# Illustration of covariance functions



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# Additions to basic tool

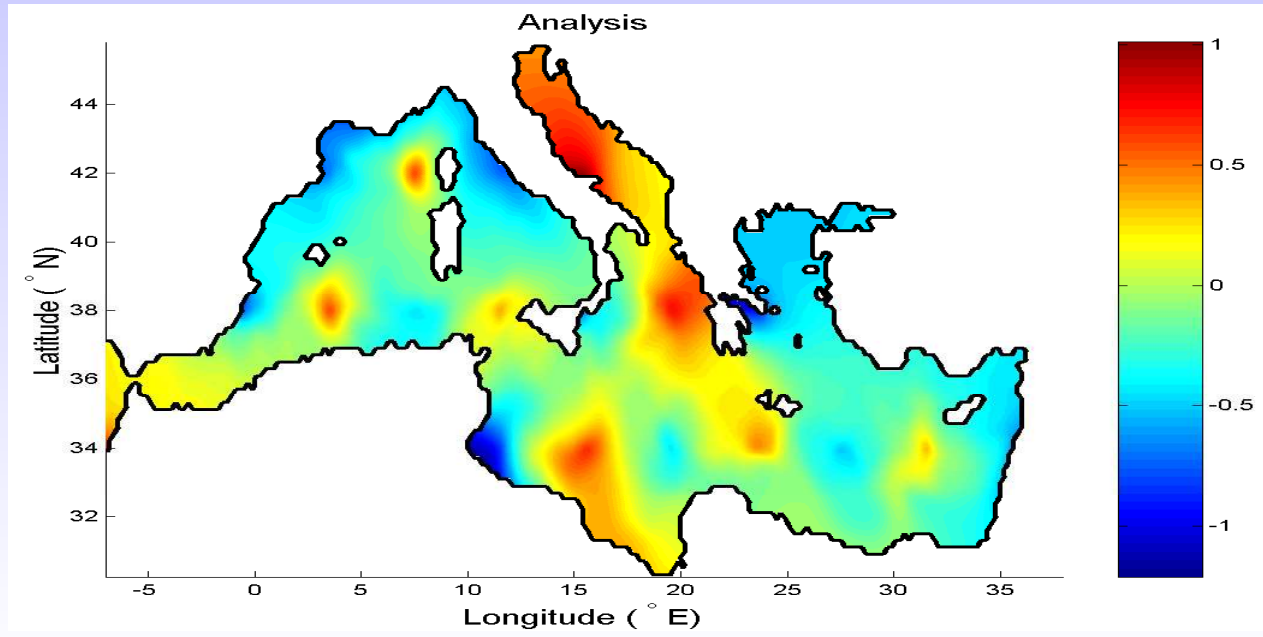
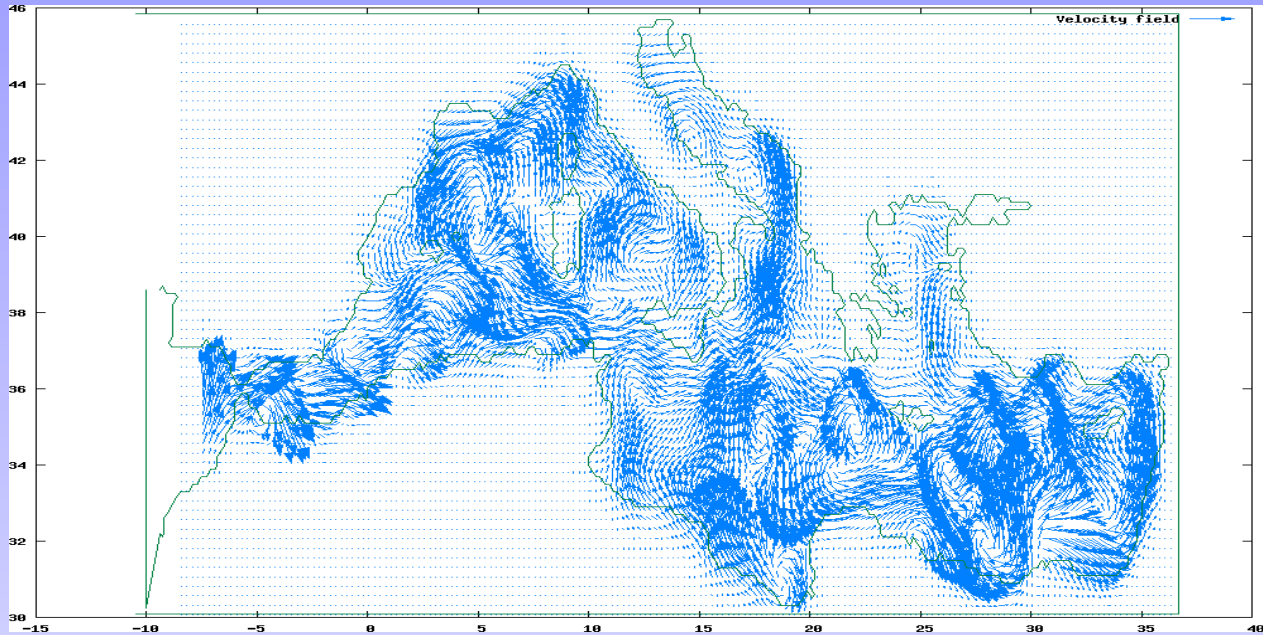
**Advection constraint:** Augmented cost function to deal with preferred correlation directions, eg, via advection with velocity  $u$  and diffusion  $\mathcal{A}$

$$J_a = J(\varphi) + \theta \int_D [\mathbf{u} \cdot \nabla \varphi - \mathcal{A} \nabla \cdot \nabla \varphi]^2 dD \quad (4)$$

## Other features

- Error fields taking data distribution into account.
- Toolbox approach allowing to design own versions.
- 3D and 4D modes by looping, hydrostatic constraint in 3D mode.
- Cross validation tools to infer statistical parameters and error estimates.
- Climatology production version with heterogeneous data distributions [\(detrending\)](#) .
- Outlier detection.
- ...

# Covariances with advection

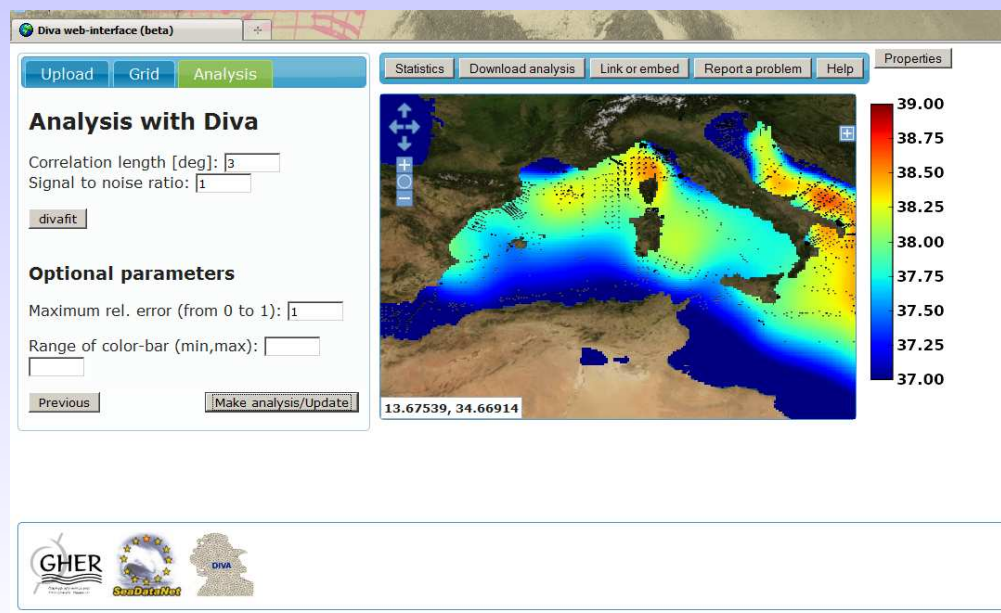
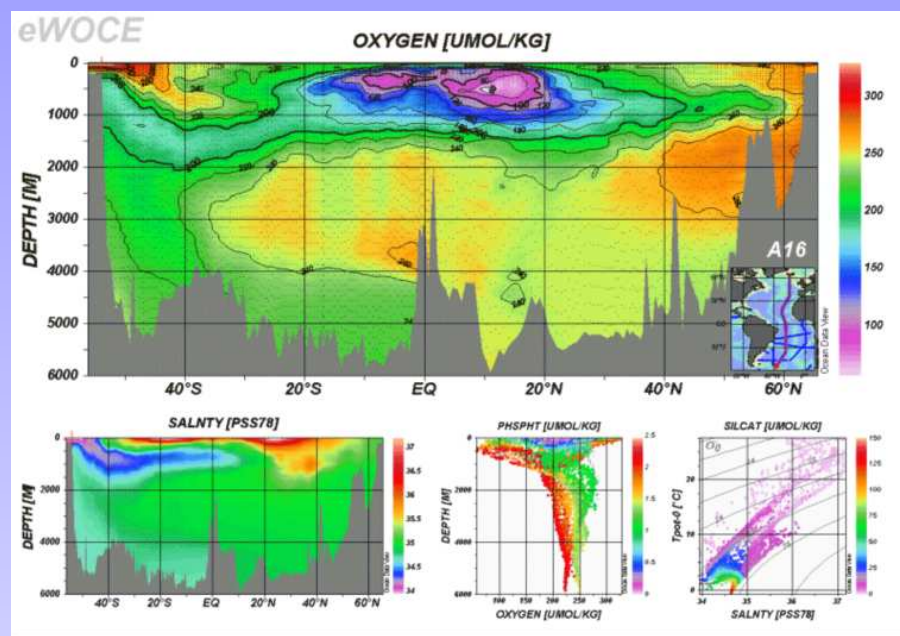
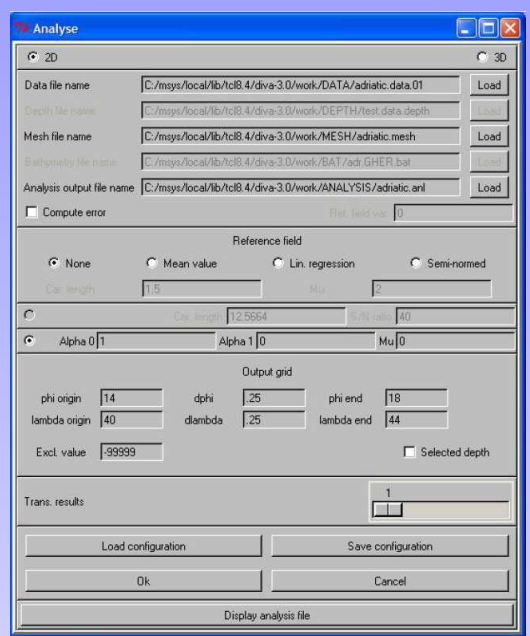


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# How to use DIVA?



```

/cygdrive/c/jmb/cd-roms/diva-4.2.1/divastripped
CALL TO STORES MODULE: IPR = 1
#####
Total nb. of pts where gridded solution is asked = 10201
Finished storing
#####
CALL TO GCUFAC MODULE: IPR = 1
#####
Trace average estimate: 0.033672668
rms of misfits 1.3200057
MAXIMUM NUMBER OF INTEGER USED: 192895
MAXIMUM NUMBER OF REAL USED: 6775828
PRIOR ESTIMATE OF INTEGER USED: 888230
PRIOR ESTIMATE OF REAL USED: 9522239
#####
D I U 0 = 4 2 = Execution Completed
#####
Output of results for user
'fort.84' -> './output/fieldgher.anl'
'fort.82' -> './output/valatxgascii.anl'
'fort.83' -> './output/fieldascii.anl'
'fort.87' -> './output/errorfieldgher.anl'
'fort.86' -> './output/errorfieldascii.anl'
'fort.71' -> './output/fielddatdatapoint.anl'
'fort.77' -> './output/gcuvval.dat'

Creation of file GridInfo.dat

'fort.87' -> './output/ghertonetcdf/fort.87'
Creating netcdf file only for field
since Uarbak and ispec are 1 0
*** SUCCESS writing NetCDF file results.nc

Analysis is finished

Becker@GHER22 /cygdrive/c/jmb/cd-roms/diva-4.2.1/divastripped
$
    
```



# DIVA on WEB

The screenshot shows the 'Diva web-interface (beta)' with three tabs: 'Upload', 'Grid', and 'Analysis'. The 'Analysis with Diva' section contains the following controls:

- Correlation length [deg]:
- Signal to noise ratio:
- 
- Optional parameters**
- Maximum rel. error (from 0 to 1):
- Range of color-bar (min,max):
- 

On the right, there are buttons for 'Statistics', 'Download analysis', 'Link or embed', 'Report a problem', and 'Help'. A 'Properties' tab is also visible. The main map displays a heatmap of the Mediterranean Sea with a color scale from 37.00 (blue) to 39.00 (red). A coordinate box at the bottom left of the map shows '13.67539, 34.66914'. A 'HOI' label is present on the left side of the map.

At the bottom of the interface, logos for GHER, SeaDataNet, and DIVA are displayed.

<http://gher-diva.phys.ulg.ac.be>  
For occasional use or quick data exploration



# DIVA on WEB

- A first prototype of DIVA on WEB was developed (not aimed as a tool for climatology production but as an easy way to access the tools for occasional uses).
- User uploads its 2D data in simple ascii format (test version now with ONE ODV4 spreadsheet support).
- Based on OpenLayer, OGC-compliant using Phyton. Maps are generated internally by a request to a server located in Liege. The request itself is prepared by a Web interface. Results are shown in OpenLayers.
- Requests could come from other servers. (ICES and VLIZ expressed interest).
- Additional outputs as netCDF files, Matlab (or Octave) files and KML files (Google Earth).

For the moment only restrictions on CPU time and data quantity.

- *Basics*
- ***When to use***
- *Critical points*
- *Examples*
- *Plans*



# Comparison



| Method      | $\min(\epsilon^2)$ | 3D    | Multivar | Ops/image         | $\epsilon(r)$ | a priori | C.V.                | anisotropy |
|-------------|--------------------|-------|----------|-------------------|---------------|----------|---------------------|------------|
| Cressman    |                    | *     | *        | $N_d N_a$         |               | $w(r/L)$ | $(L)$               | $(*)$      |
| O.I.        | *                  | *     | *        | $N_d^3 + N_d N_a$ | *             | $c(r/L)$ | $L, \sigma^2/\mu^2$ | $(*)$      |
| <b>DIVA</b> | *                  | $(*)$ | $(*)$    | $N_e^{5/2}$       | *             | $K(r/L)$ | $L, \sigma^2/\mu^2$ | *          |
| DINEOF      | $(*)$              | *     | *        | $N_a^{5/4}$       | $(*)$         | stat.    | $N$                 | *          |

$N_d$  : number of data points

$N_a$  : number of grid points for analysis

$N_e$  : number of finite elements

$N$  : number of EOFs

$L$  : correlation length

$\sigma^2/\epsilon^2$  : signal to noise ratio

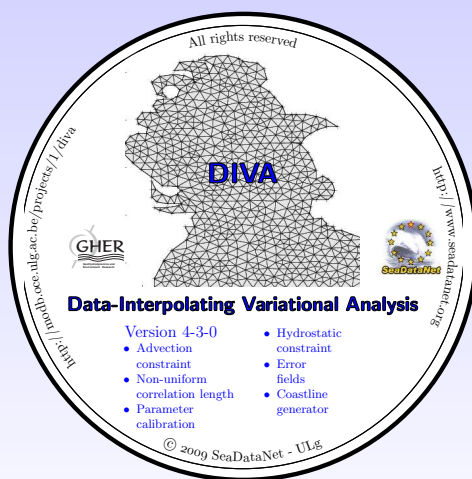
\* : available feature

$(*)$  : available with some adaptations

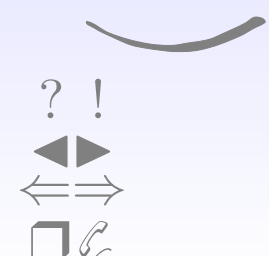


# Reference tool for SeaDataNet:

- Large data sets expected
- Error fields requested
- GUI, ODV-interface, WEB-interface (beginners) or scripts (expert mode)
- 3D (= stacking of 2D)
- G.C.V. for calibration of  $L$  and  $S/N$ .
- Land and underwater islands/obstacles taken into account (covariance changed).



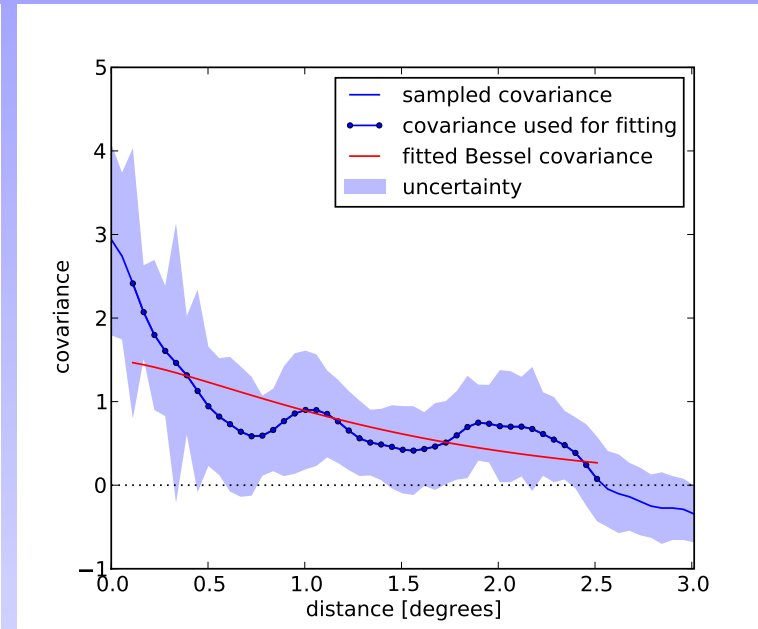
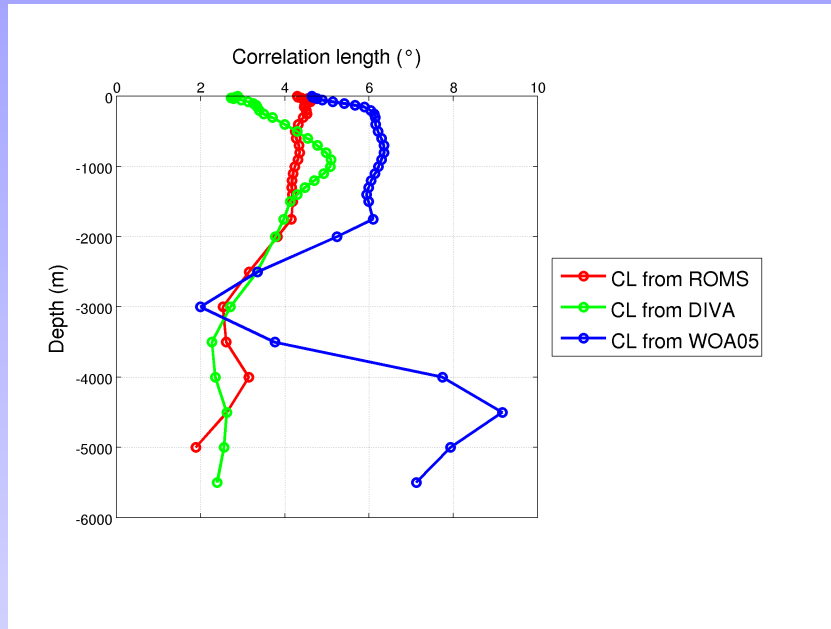
<http://modb.oce.ulg.ac.be/projects/1/diva>



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# Parameter calibration



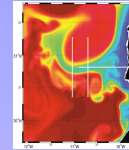
Spatial coherence of parameters: here correlation length obtained with covariance fitting (Troupin *et al.*, 2010).





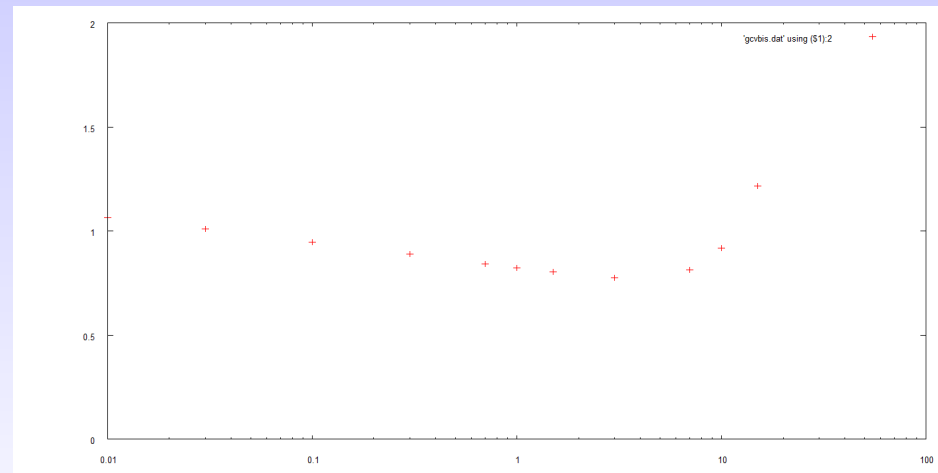
# Signal to noise ratio

The most elusive parameter.



- Noise is not only instrumental error:
- Very hard problem to decide on value with dependent data (cross-validation approaches fail).

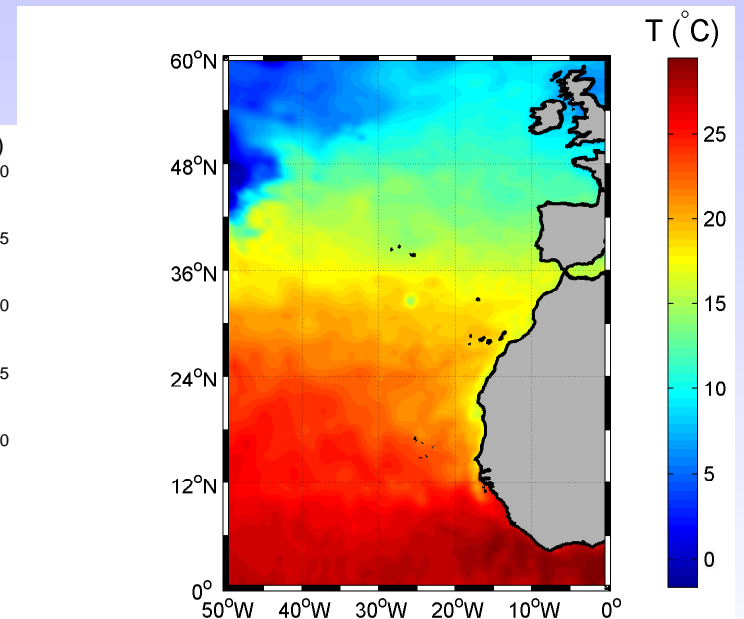
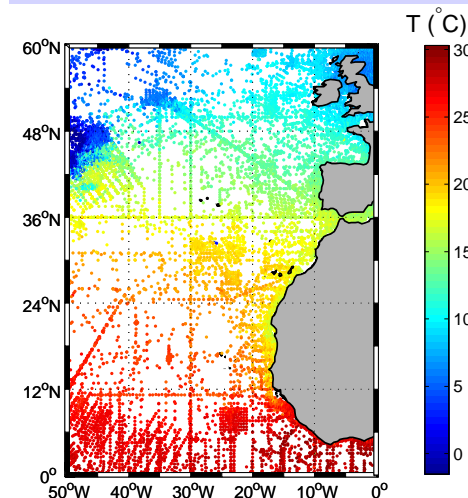
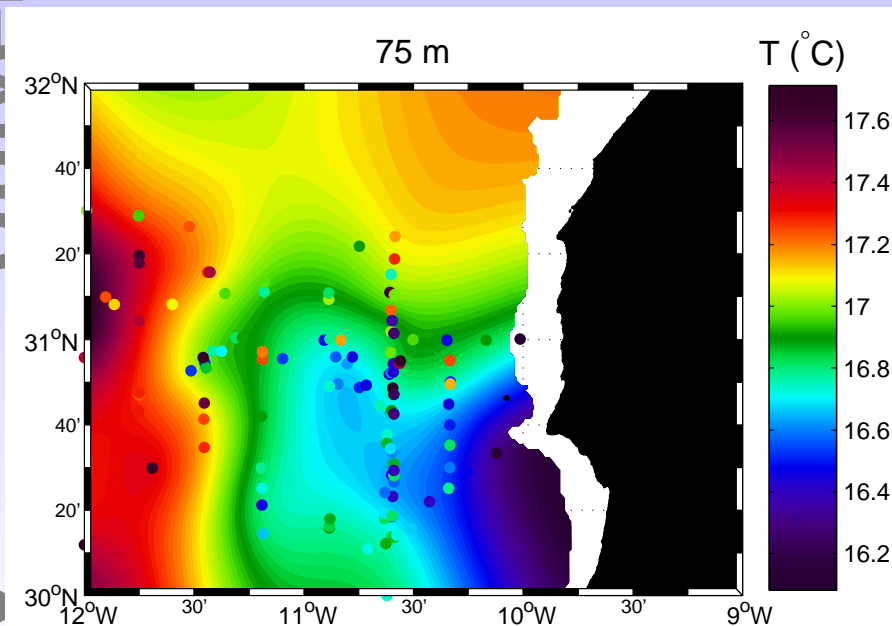
A series of estimation tools are provided with DIVA, but here the experience of oceanographers is critical. A posteriori analysis of residuals allows to verify coherence. With reasonable amount of data, parameter not critical for analysis but for error estimates.



# Data availability

## THE problem.

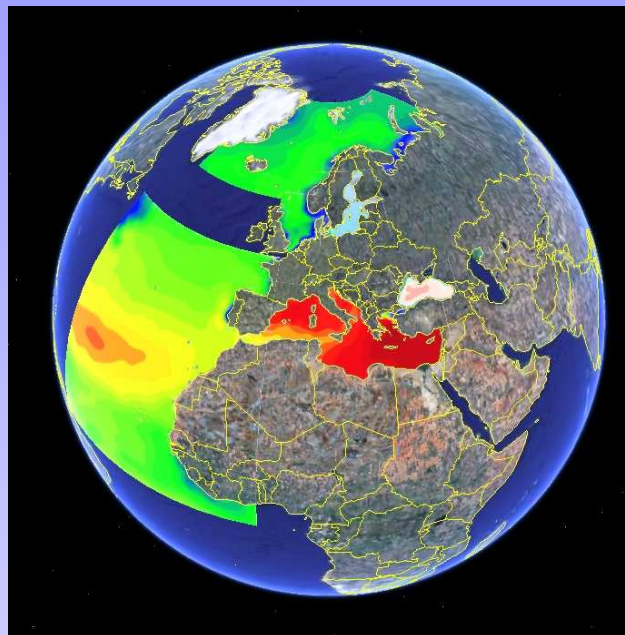
- "Enough data" different for cruise snapshots or climatologies
- Recurrent discussion: if and how to mask analyses. Scientific approach (provide field and error estimate) vs large public approach (danger of misinterpretation or misuse).
- Validation and QC of products (independent data, other climatologies).



- *Basics*
- *When to use*
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# Regional products within SeaDataNet

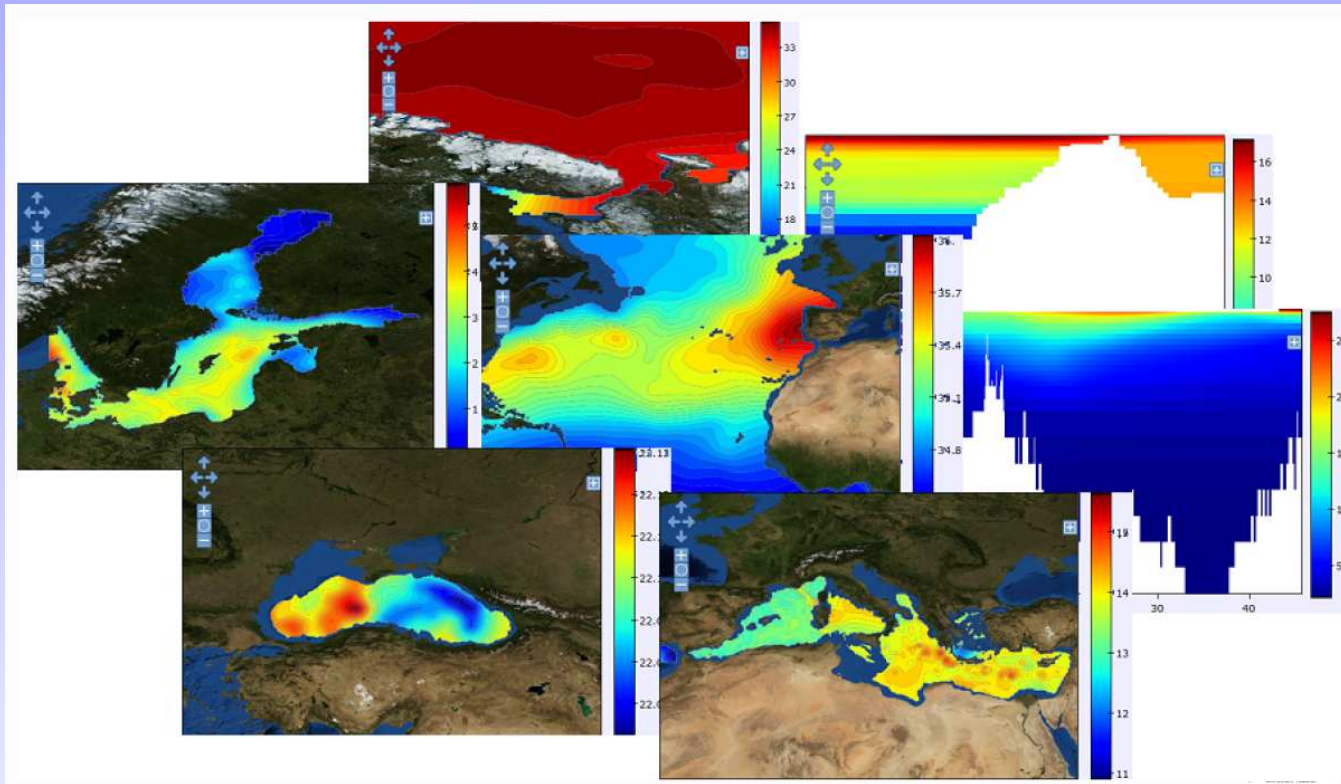


Installation of latest DIVA version and additional climatology-production tools.



# Climatology on WEB

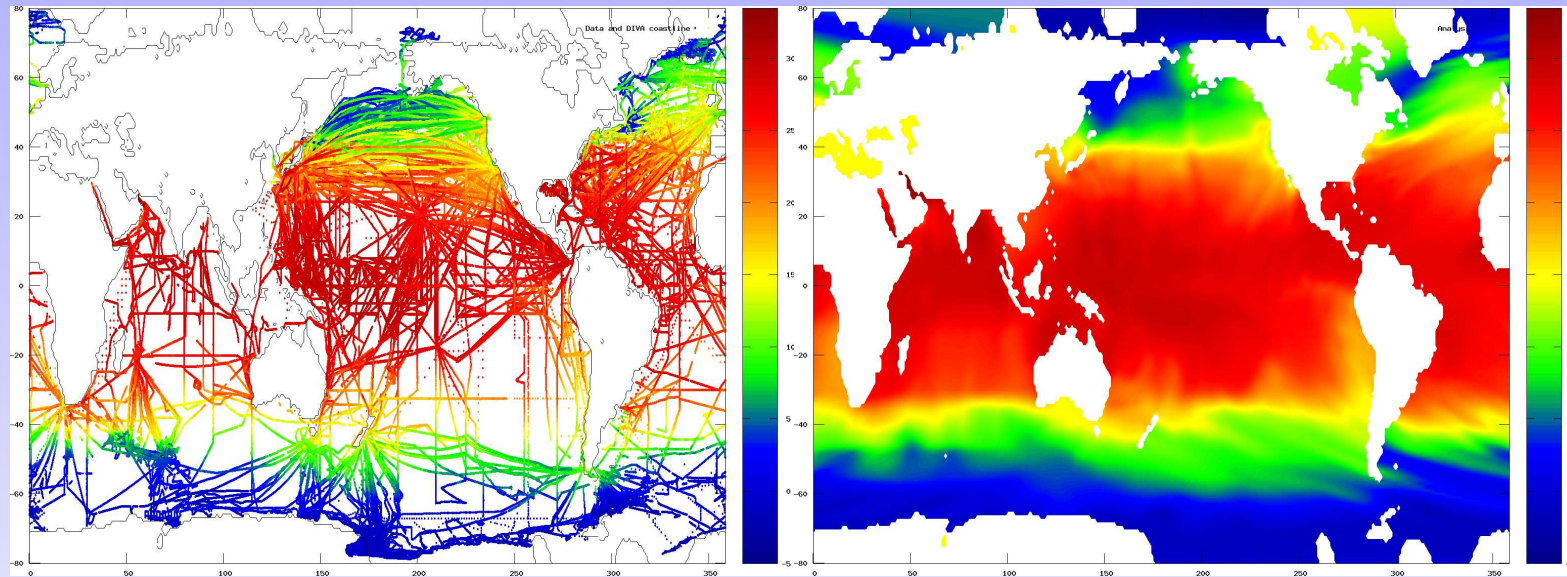
Regional products uniformly and automatically prepared as 4D netCDF files. Hence possibility for common interface:



<http://gher-diva.phys.ulg.ac.be>  
Includes vertical transect tool.

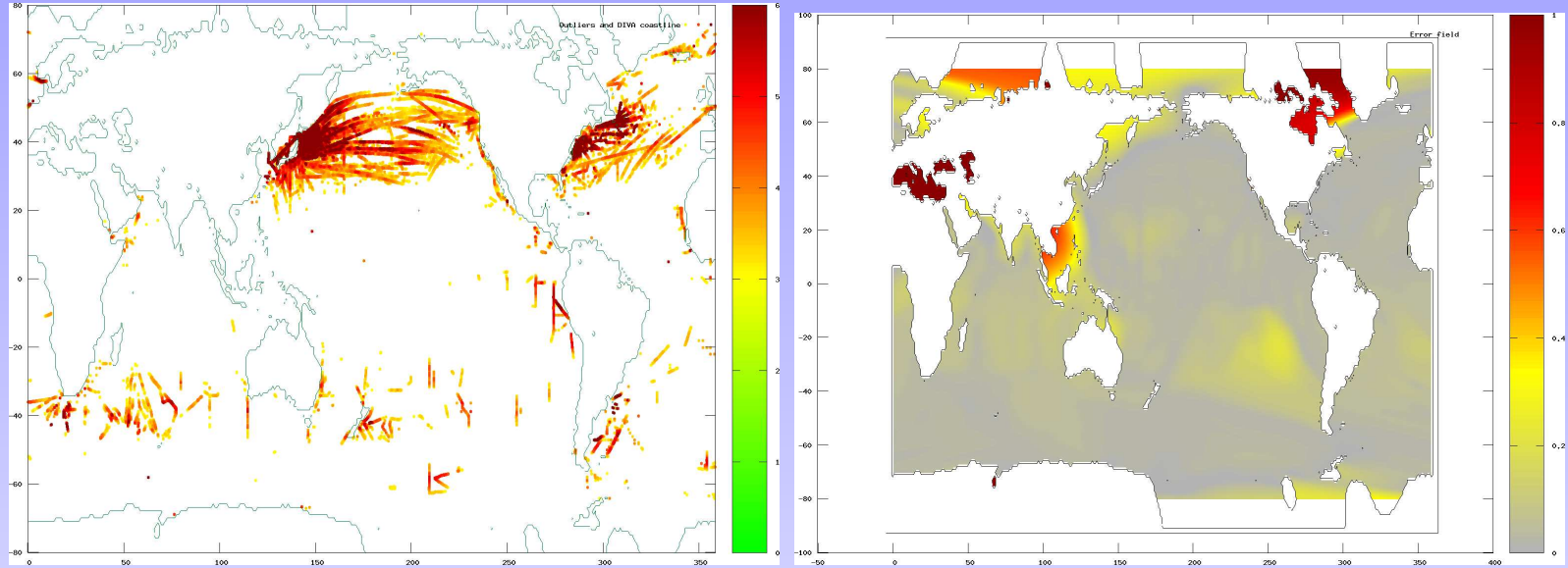
# Huge problems

LDEO data base with  $4.5 \cdot 10^6$  measurements (Takahashi *et al.*, 2009). Running on a laptop within a few minutes. Shown here, temperature fields.

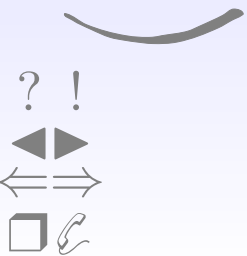


# Huge problems, outliers and relative error field

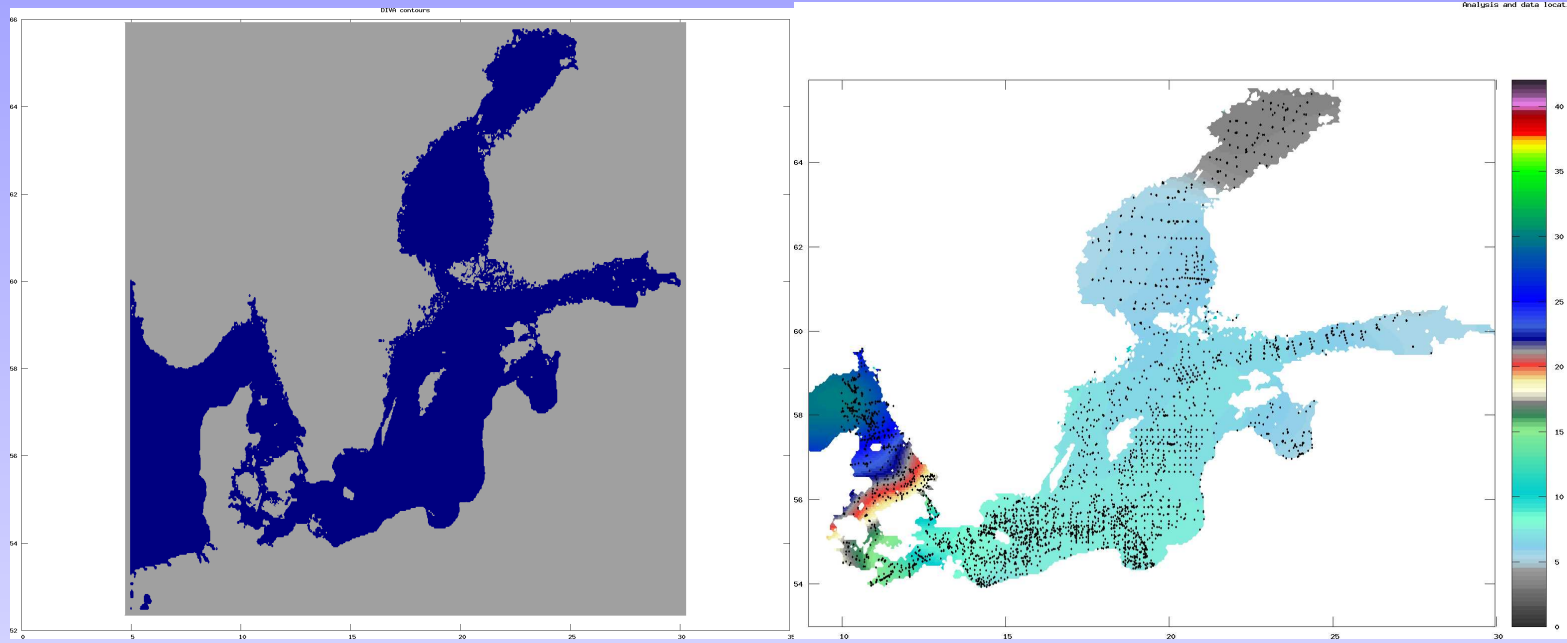
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Outliers detected via comparison of statistically expected residuals (value provided by the DIVA analysis) and actual residuals.



# Heterogeneous case



Baltic Salinity Climatology (Bassompierre *et al.*, 2010)

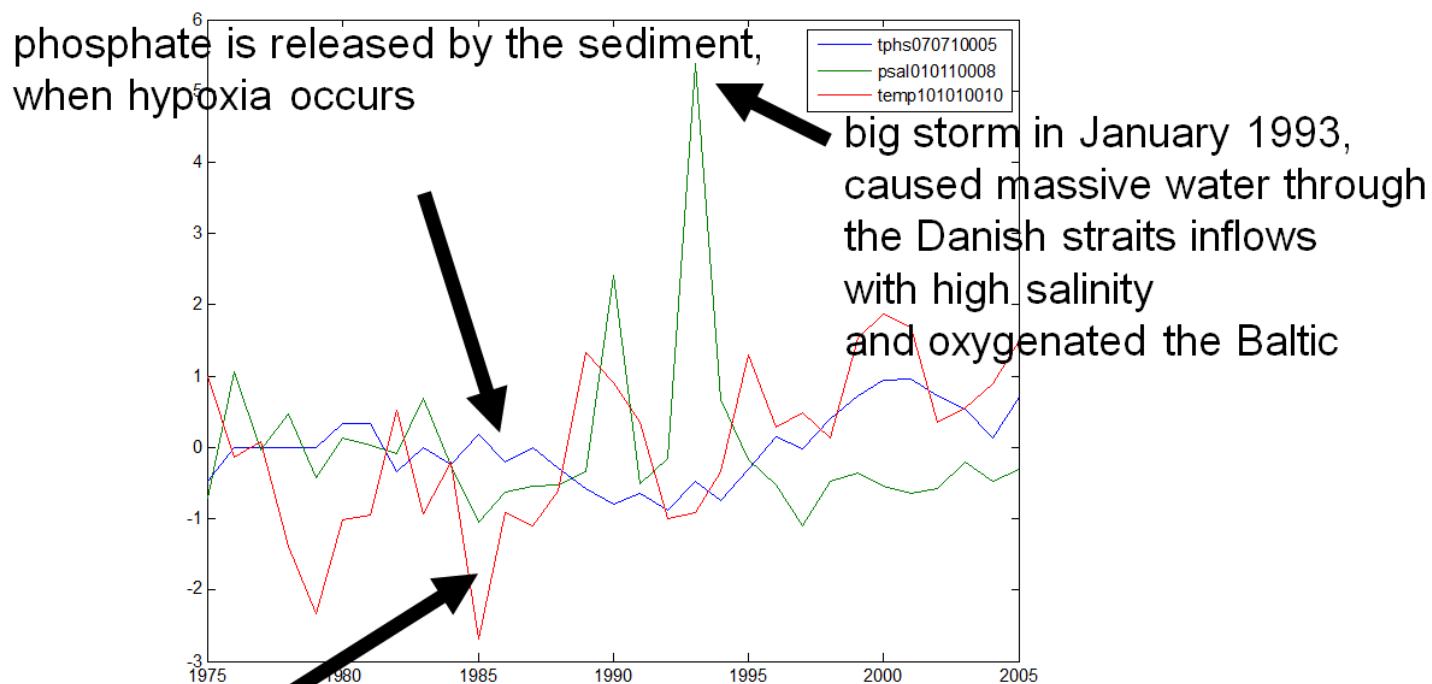
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# Heterogeneous case, trends

X= nutrient & climatic trends from Kattegat to Bothnian Bay (SDN products)



tphs0707005= total phosphate July -100 m  
psal 0101108=salinity January level -40 m  
temp101010010=temperature October -20 m

(Bassompierre *et al.*, 2010)



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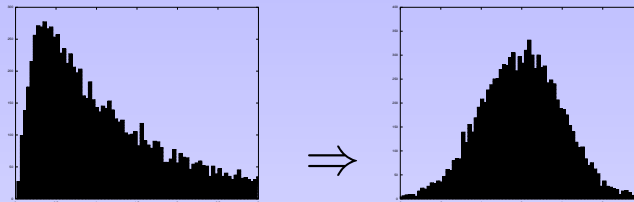
# Computational aspects

- Migration to F95 with dynamic memory allocation.
- Open source tools for large matrix calculations and parallel machines. (But best parallelization remains on parameters, seasons ...)
- Feeding of very large data arrays ( $10^7$ ).
- Web visualization (both single DIVA analysis and 4D netCDF files).

In all cases, backward compatibility, portability and independence on proprietary software

# Functional aspects

- Common problem in SeaDataNet and other projects: lack of biochemical data, strange statistical distributions.
  - ★ Multivariate approach with non-collocated data.
  - ★ Data transformation tools including anamorphosis.
  - ★ Absence-presence data for probability analysis.



- Adapting input possibilities.



- Investigate solutions to calibration with dependent data.

# Very profound changes?

- N-Dimensional generalization. Easy in finite differences, awful in FEM, with need to rewrite completely the code.
- More complicated physical laws (source terms, vertical advection...).
- Diva-on-web to be used from within CDI interface for analyses on the fly ? (Including possibility to process restricted data without actually delivering them?).
- Rethinking of folder structure for better support of multiple users in a single installation.

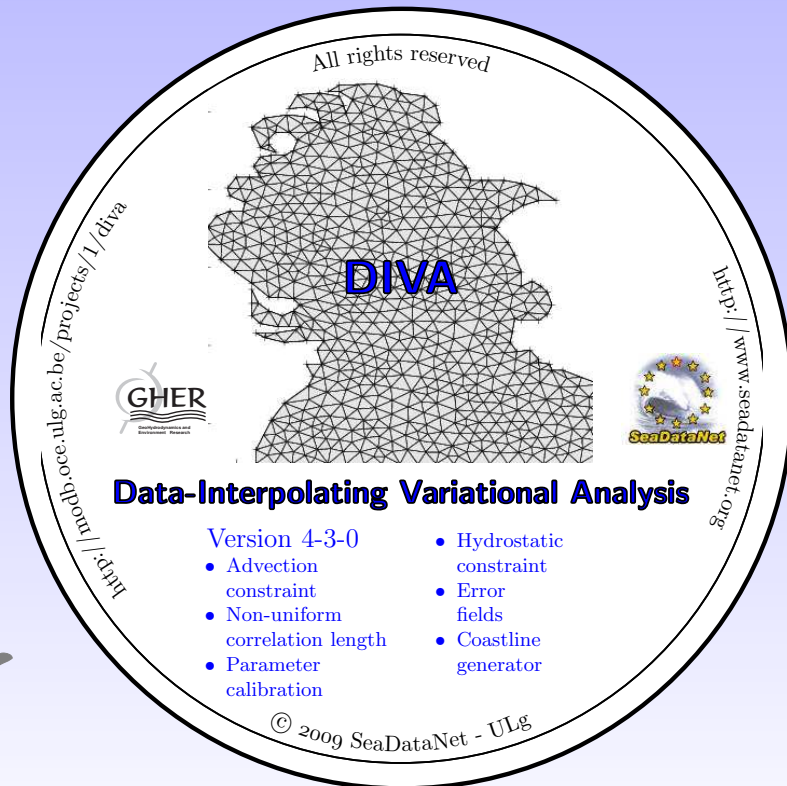
Some kind of conservatism here: robustness is priority ( $10^2$  to  $10^4$  spatial analyses for a 4D product). Funding ?

## *Proud of*

- SeaDataNet regional centers now well working with DIVA4D.
- Group that was most sceptical at Athens SDN annual meeting now the most happy user.
- DIVA development nicely user driven (annual workshop).
- ODV4 spreadsheet format gaining acceptance and easing up operations.
- DIVA incorporation into ODV very stable.
- Product preparation very effective at spotting outliers (visually or by automated a posteriori outlier detection) and testing data flow.

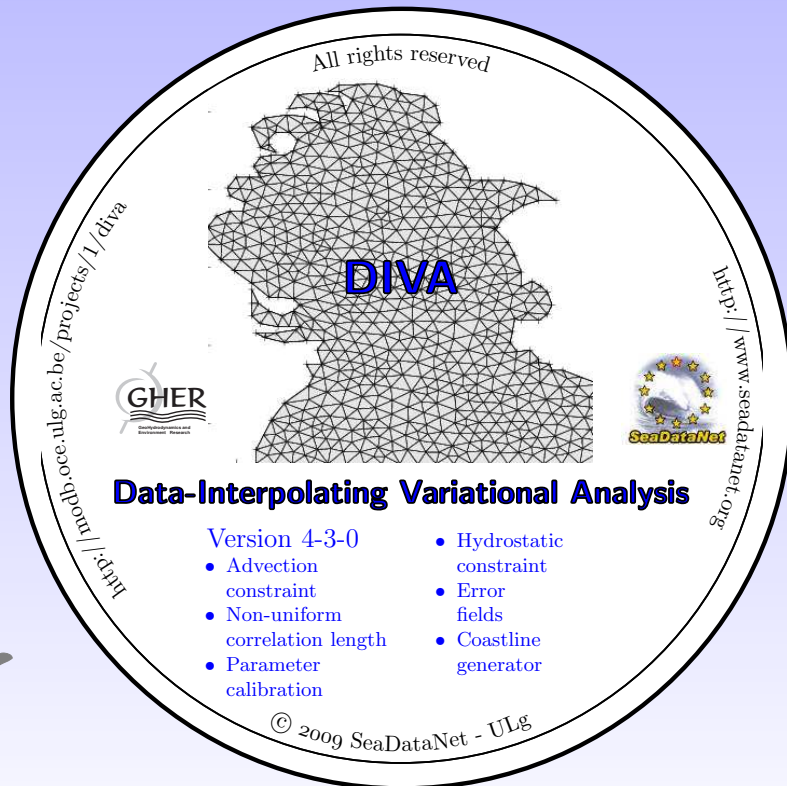
# Acknowledgements

Special thanks to Helge Sagen, Marina Tonani, Sissy Iona, Cristina Tronconi, Jacob Carstensen, Marc Bassompierre, Serge Scory, Øivind Østensen, Pierre Brasseur, Jean-Michel Brankart and Michel Rixen for comments on DIVA evolution. Reiner Schlitzer designed and implemented the ODV coupling. SeaDataNet financed the continued development. The support from the Fonds pour la Formation la Recherche dans l'Industrie et dans l'Agriculture (FRRIA) and National Fund for Scientific Research is also acknowledged.



# Acknowledgements

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P. Brasseur, J.-M. Beckers, J.-M. Brankart, and R. Schoenauen. Seasonal temperature and salinity fields in the Mediterranean Sea: Climatological analyses of a historical data set. *Deep Sea Research*, 43:159–192, 1996.



Ch. Troupin, F. Machin, M. Ouberdous, D. Sirjacobs, A. Barth, J.-M. Beckers. High-resolution Climatology of the North-East Atlantic using Data-Interpolating Variational Analysis (Diva). *Journal of Geophysical Research*, accepted, 2010.



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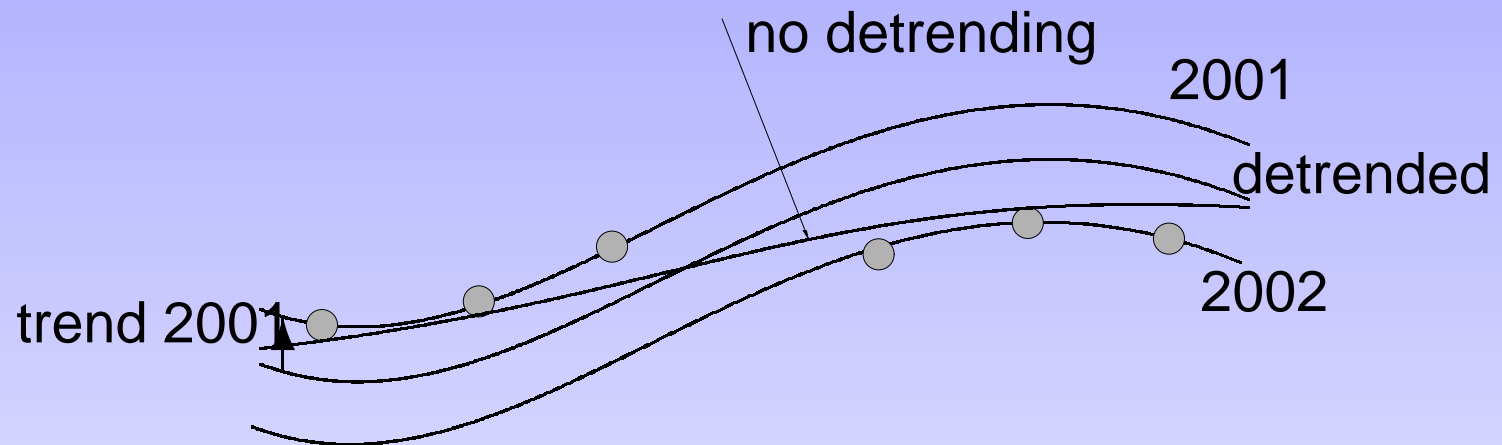


M. Rixen, J.-M. Beckers, S. Levitus, J. Antonov, T. Boyer, C. Maillard, M. Fichaut, E. Balopoulos, S. Iona, H. Dooley, M.-J. Garcia, B. Manca, A. Giorgetti, G. Manzella, N. Mikhailov, N. Pinardi, M. Zavatarelli, and the Medar Consortium. The Western Mediterranean Deep Water: a proxy for global climate change. *Geophysical Research Letters*, 32, 2005.



# Detrending

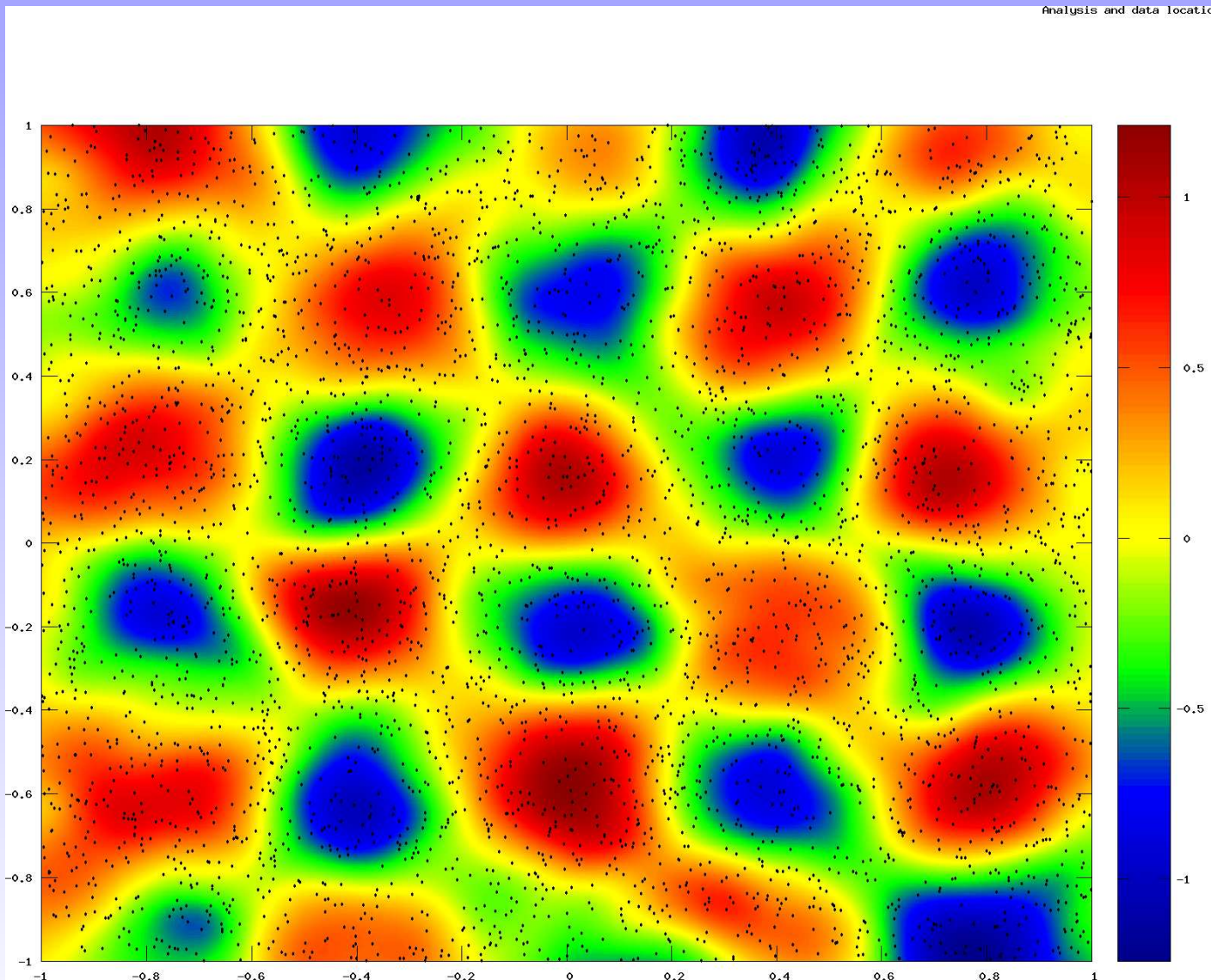
Heterogeneous data distribution:



First analysis show a bias for each year's data with respect to first analysis. Subtract the bias estimate and redo the analysis, accumulating the bias. After convergence, detrended analysis+bias of the year.

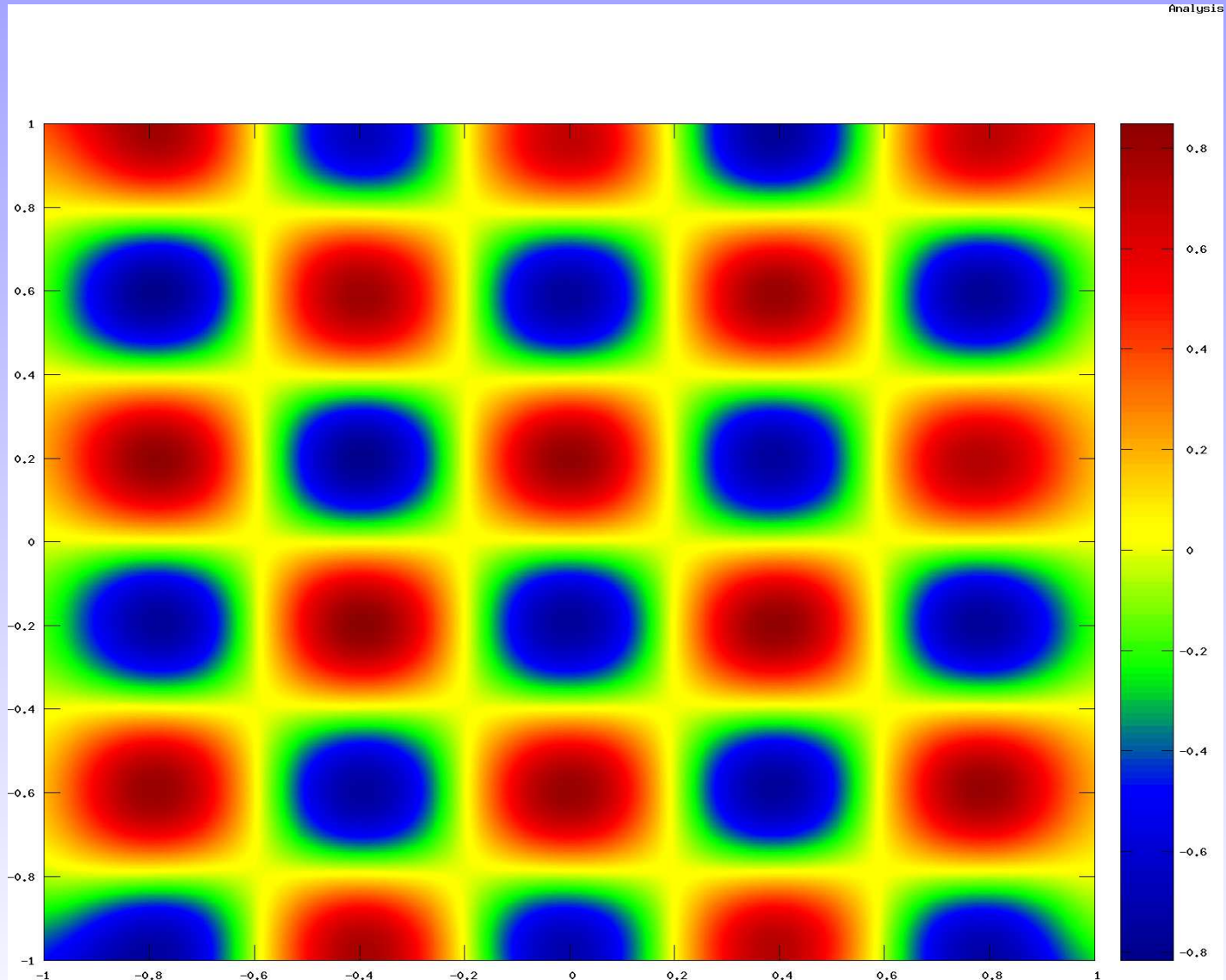


# Example without detrending



DIVA analysis of sin-cosine spatial structure with superimposed decadal, seasonal and daily cycles and noise.

# Example with detrending



DIVA analysis of sin-cosine spatial structure with superimposed decadal, seasonal and daily cycles and noise.

# Trends can also be retrieved

